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# The application of the relaxation method to the solution of problems involving the flow of fluids through porous media

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# THE APPLICATION OF THE RELAXATION METHOD TO THE SOLUTION OF PROBLEMS INVOLVING THE FLOW OF FLUIDS THROUGH POROUS MEDIA

BY

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A

THESIS

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# PREFACE

The purpose of this investigation is to determine the feasibility of applying the relaxation method of mathematical analysis to the solution of problems in rheology. The relaxation method was first introduced by R. V. Southwell[1] who applied it to the problem of electrical conducitivity. It was later used by M. W. Emmons[2] to solve two-dimensional heat conduction problems.

The steady-state flow of fluids through porous media can be difficult or impossible by the usual analytical methods even with rather simple boundary conditions. The relaxation method promises to provide an easier method of solution and to provide a means of solving some of the problems not yet solved. It is the object of this thesis to establish the feasibility of the relaxation method rather than to solve any new problem. It finds a maximum usefulness in reservoir mechanics of petroleum engineering.

(1) Southwell, R. V., Proc. Royal Society, Series A, Vol. 168, pp. 317-350 (1938)

(2) Emmons, H. W., Transactions A.S.M.E., Vol. 63, No. 6, pp. 607-615 (Aug. 1943)

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# INTRODUCTION

Before illustrating the use of the relaxation method, the correspondence between heat conduction and the flow of an incompressible liquid through a porous media will be shown; since the problem of applying the relaxation method to flow of fluids was attempted when it was seen that M. W. Emmons[3] has applied it successfully to heat conduction problems.

| Heat Conduction | Steady State Flow |
|---|---|
| Temperature T | Pressure P |
| Thermal Conductivity k | $\frac{\text{Permeability}}{\text{Viscosity}}$ $\frac{k}{u}$ |
| Rate of Heat Transfer $Q = -kA \frac{dT}{dx}$ | Velocity Vector $V = - \frac{k}{u} \frac{dP}{dx}$ |
| Isothermal Surface T = Constant | Equipressure Surface P - Constant |

From above comparison it is seen that Fourier's Law is similar to D'Arcy's Law, the only difference being that temperature is substituted by pressure.

Also from Laplace's equation, in heat conduction:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

and in fluid flow, introducing the velocity potential $\phi$:

$$\frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^2 \phi_y}{\partial y^2} + \frac{\partial^2 \phi_z}{\partial z^2} = 0$$

(3) Emmons, op. cit. p. iii.

Therefore, since the similarity between heat conduction and fluid flow is clearly shown, it is concluded that the relaxation method will also apply to problems involving fluid flow.

The main idea in solving steady fluid flow problems by the relaxation method is that fluid mass in any closed system can be neither created nor destroyed. In other words at steady flow conditions the total quantity, Q, of fluid at any given point at any given instant of time must add up to zero e.g. as much fluid is flowing towards the point as away from it.

The above idea is the basis for the solution of fluid problems involving steady state conditions. Essentially the procedure followed is to assume values of pressure at all points whose pressure is to be determined, and to calculate Q at any particular point using D'Arcy's Law. For steady flow $Q = 0$, so reassign values of pressure until Q approaches very closely or equals zero.

Four problems involving radial and rectangular flow actually show the feasibility of using the relaxation method to solve problems of similar nature. The radial flow problems can be actually checked by formula as will be shown subsequently.

PROBLEM 1

SOLUTION OF SIMPLE RADIAL FLOW PROBLEM

USING THE RELAXATION METHOD OF

MATHEMATICAL ANALYSIS

As an illustration of the use of the Relaxation Method, consider the following two-dimensional problem.

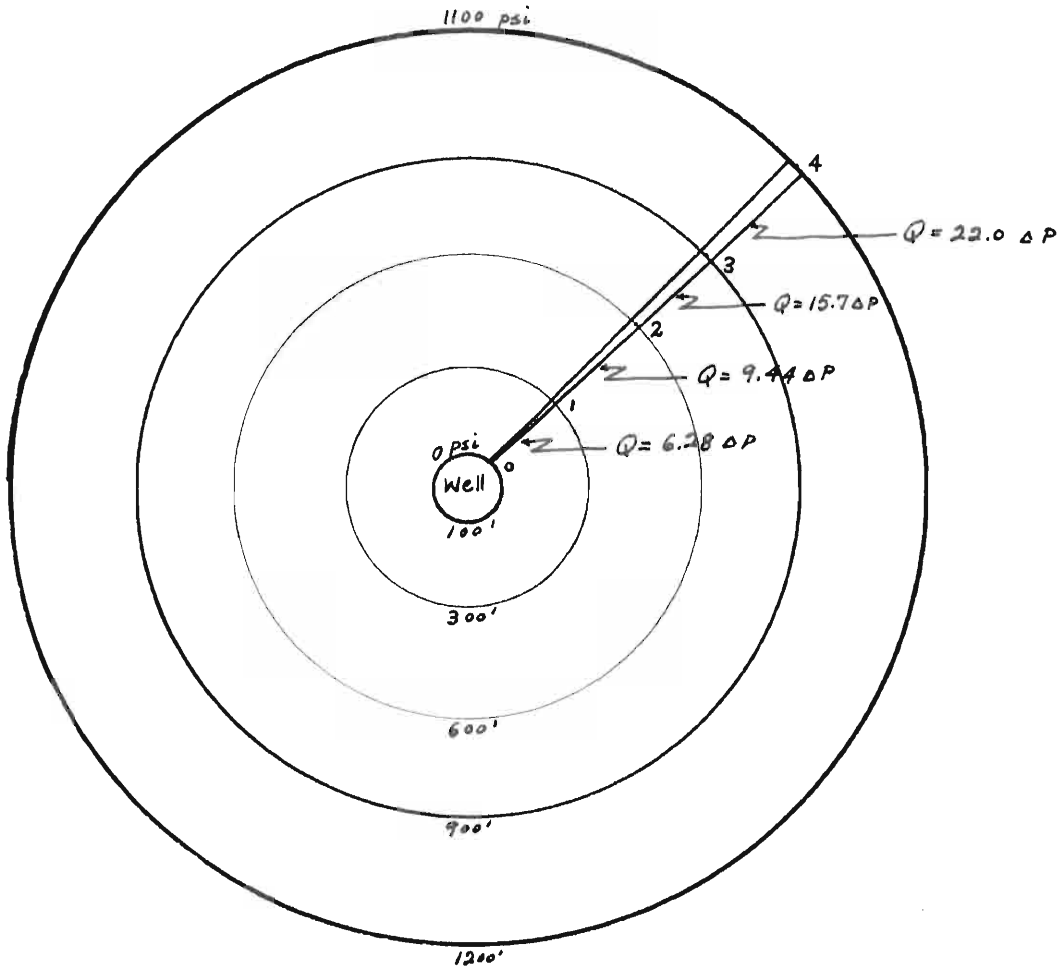



Fig. 1

Given: well bore 100 feet
pressure 0 psi

drainage radius 1200 feet
pressure 1100 psi

To find pressure at radius of 300', 600', and 900'.

1. Assume straight line variation of pressure with drainage radius.

2. Flow is radial only.

3. Since drainage area is symmetrical in all directions, take a sample equivalent pipe on any radius as shown above.

From D'Arcy's Law

$$Q = -\frac{k}{u} A \frac{\Delta P}{\Delta R}$$

where k = permeability

u = viscosity

A = cross-sectional area of flow

$\Delta P$ = difference in pressures

$\Delta R$ = difference in radiuses along flow lines

let $\frac{k}{u} = 1$ for particular problem

$A \sim R$  R = arithmetic average value of radius

$A = 2\pi R$ (for unit thickness)

$$Q = \frac{2\pi R}{\Delta R} \Delta P$$

For particular problem

$$_0Q_1 = {}_{100'}Q_{300'} = \frac{2\pi R}{\Delta R}\Delta P = \frac{2\pi(200)}{200}\Delta P = 6.28\,\Delta P$$

$$_1Q_2 = {}_{300'}Q_{600'} = \frac{2\pi(450)}{300}\Delta P = 9.44\,\Delta P$$

$$_2Q_3 = {}_{600'}Q_{900'} = \frac{2\pi(750)}{300}\Delta P = 15.7\,\Delta P$$

$$_3Q_4 = {}_{900'}Q_{1200'} = \frac{2\pi(1050)}{300}\Delta P = 22.0\,\Delta P$$

as shown on Fig. 1

From plate No. 1 assuming straight line variation of pressure with drainage radius,

P @ 100' = 0 psi

P @ 300' = 200 psi

P @ 600' = 500 psi

P @ 900' = 800 psi

For steady flow conditions, total Q = 0 at any point.

Calculate Q with assumed pressures at particular points, and if $Q \neq 0$ assume other values of pressure until $Q = 0$ with correct values of pressure.

a.) At point where r = 300'

Flowing in (+) $Q_{300\ 600} = 9.44\ (500-200) = +\ 2832$ units

Flowing out (-) $Q_{100\ 300} = 6.28\ (200-0) = -1256$ units

Total $Q = +2832-1256 = +\ 1576$ units

b.) At point where r = 600'

Flowing in (+) $Q_{600\ 900} = 15.7\ (800-500) = +4710$ units

Flowing out (-) $Q_{300\ 600} = 9.44\ (500-200) = -2832$ units

Total $Q = +4710-2832 = +\ 1878$ units

c.) At point where r = 900'

Flowing in (+) $Q_{900\ 1200} = 22.0\ (1100-800) = +\ 6600$ units

Flowing out (-) $Q_{600\ 900} = 15.7\ (800-500) = -4710$ units

Total $Q = +6600-4710 = +1890$ units

From above it is seen that assumed values of pressure are incorrect for $Q \neq$ zero for all three points, as Q is not zero for any point.

Therefore using the method of relaxation, that is continually assuming values of pressures for a point until equivalent Q values are equal to or approach zero.

This is shown on table No. 1 where all results are tabulated in their order of calculation.

To check results obtained, compare with values of pressure calculated from the formula for radial flow to a sink as shown on page 10.

## Calculations

Given $P_0 = 0$ psi

$P_1 = 200$ psi

$P_2 = 500$ psi

$P_3 = 800$ psi

$P_4 = 1200$ psi

Step 0

$Q_1 = 9.44\ (500-200)\ -6.28\ (200-0) = +1570$

$Q_2 = 15.7\ (800-500)\ -9.44\ (500-200) = +1880$

$Q_3 = 22.0\ (1100-800)\ -15.7\ (800-500) = +1890$

Step I

Change pressure @3 and make it 800+ 100 = 900 psi

$Q_3 = 22.0\ (1100-900)\ -15.7\ (900-500) = -1880$

$Q_2 = 15.7\ (900-500)\ -9.44\ (500-200) = 3450$

Step II

Change pressure @2 and make it 500+ 200 = 700 psi

$Q_2 = 15.7\ (900-700)\ -9.44\ (700-200) = -1588$

$Q_1 = 9.44\ (700-200)\ -6.28\ (200-0) = +3468$

$Q_3 = 22.0\ (1100-900)\ -15.7\ (900-700) = +1260$

Step III

Change pressure @1 and make it 200+ 200 = 400 psi

$Q_1 = 9.44\ (700-400)\ -6.28\ (400-0) = +318$

$Q_2 = 15.7\ (900-700)\ -9.44\ (700-400) = +302$

Step IV

Change pressure @3 and make it 900 + 50 = 950 psi

$Q_3$ = 22.0 (1100-950) -15.7 (950-700) = -635

$Q_2$ = 15.7 (950-700) -9.44 (700-400) = +1087

Step V

Change pressure @2 and make it 700 + 80 = 780 psi

$Q_2$ = 15.7 (950-780) -9.44 (780-400) = -824

$Q_1$ = 9.44 (780-400) -6.28 (400-0) = +1087

$Q_3$ = 22.0 (1100-950) -15.7 (950-780) = + 621

Step VI

Change pressure @3 and make it 950 + 20 = 970 psi

$Q_3$ = 22.0 (1100-970) -15.7 (970-780) = -133

$Q_2$ = 15.7 (970-780) -9.44 (780-400) = -510

Step VII

Change pressure @1 and make it 400 + 70 = 470 psi

$Q_1$ = 9.44 (780-470) -6.28 (470-0) = -27

$Q_2$ = 15.7 (970-780) -9.44 (780-470) = + 150

Step VIII

Change pressure @2 and make it 780 + 8 = 788 psi

$Q_2$ = 15.7 (970-788) -9.44 (788-470) = -51

$Q_3$ = 22.0 (1100-970) -15.7 (970-788) = -7

$Q_1$ = 9.44 (785-470) -6.28 (470-0) = + 48

Step IX

Change pressure @1 and make it 470 + 4 = 474 psi

$Q_1 = 9.44\ (788-474) - 6.28\ (474-0) = +15$

$Q_2 = 15.7\ (970-788) - 9.44\ (788-474) = +13$

Step X

Change pressure @2 and make it 788 + 1 = 789 psi

$Q_2 = 15.7\ (970-789) - 9.44\ (789-474) = -12$

$Q_1 = 9.44\ (789-474) - 6.28\ (474-0) = +24$

$Q_3 = 22.0\ (1100-970) - 15.7\ (970-789) = +8$

Step XI

Change pressure @1 and make it 474 + 2 = 476 psi

$Q_1 = 9.44\ (789-476) - 6.28\ (476-0) = -7$

$Q_2 = 15.7\ (970-789) - 9.44\ (789-476) = +7$

Tabulated Results

| Step | $Q_0$ | $P_0$ | $Q_1$ | $P_1$ | $Q_2$ | $P_2$ | $Q_3$ | $P_3$ | $Q_4$ | $P_4$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | 0 | +1570 | 200 | +1880 | 500 | +1890 | 800 | | 1100 |
| 1 | | | | | +3450 | | -1880 | +100 | | |
| 2 | | | +3468 | | -1588 | +200 | +1260 | | | |
| 3 | | | +318 | +200 | +302 | | | | | |
| 4 | | | | | +1087 | | -635 | +50 | | |
| 5 | | | +1073 | | -824 | +80 | +621 | | | |
| 6 | | | | | -510 | | -133 | +20 | | |
| 7 | | | -27 | +70 | +150 | | | | | |
| 8 | | | +48 | | -51 | +8 | -7 | | | |
| 9 | | | +15 | +4 | +13 | | | | | |
| 10 | | | +24 | | -12 | +1 | +8 | | | |
| 11 | | | -7 | +2 | +7 | | | | | |
| Final Pressure | | 0psi | | 476psi | | 789psi | | 970psi | | 1100psi |

Table 1

It is seen from plate No. 1 that the relaxation method applied to problem No. 1 resulted in getting values of pressures for points 1, 2, and 3 very close or identical to those calculated from formula (1) for radial flow to a well.

$$P = \frac{P_e - P_w}{\ln R_e/R_w} \ln \frac{R}{R_w} + P_w \qquad (1)$$

where $P$ = pressure @ radius R

$P_e$ = known pressure @ radius $R_e$
= 1100 psi

$P_w$ = pressure at face of well bore
= 0 psi

$R_e$ = drainage radius where pressure is known
= 1200 feet

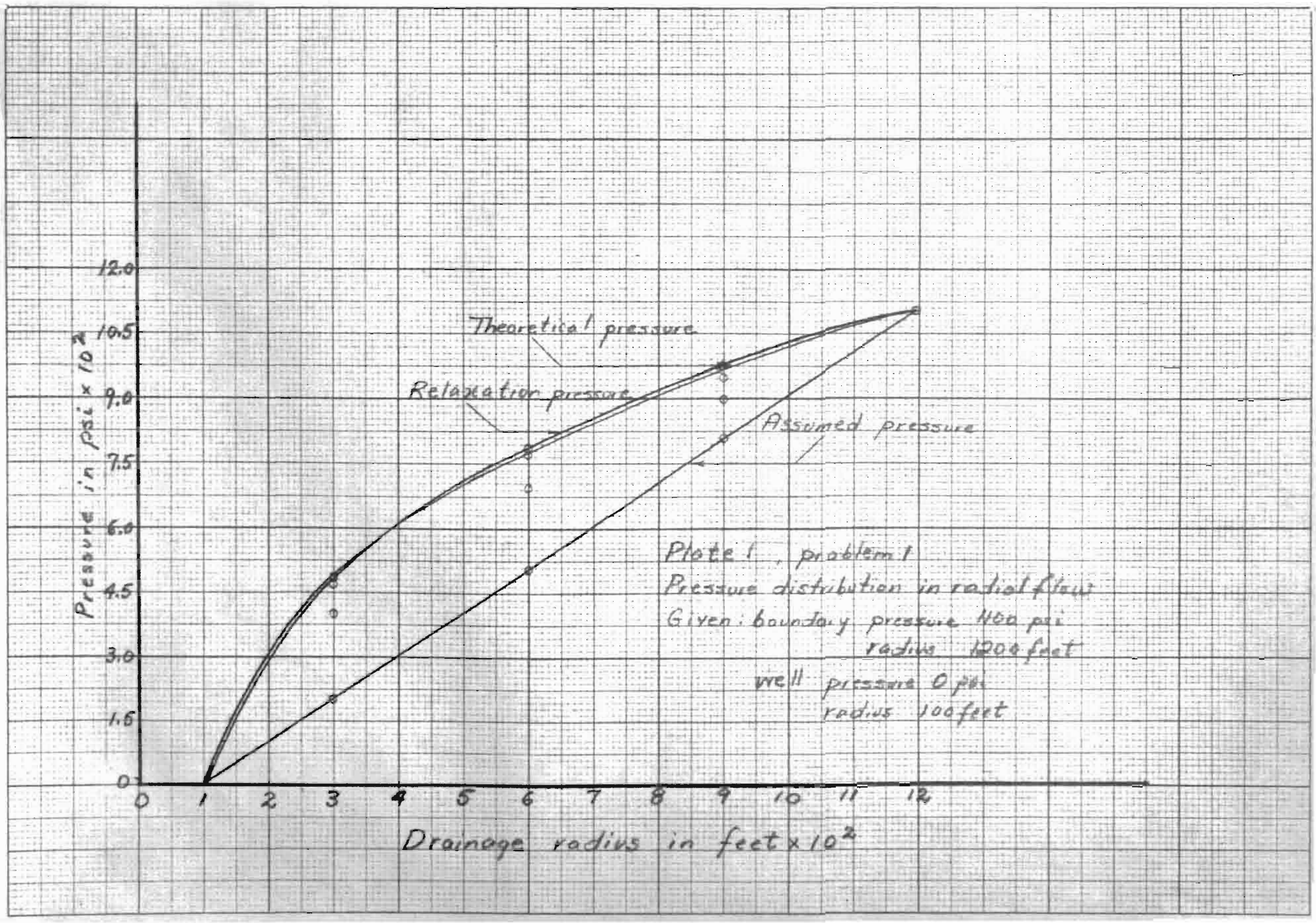
$R_w$ = radius of well
= 100 feet

$$P = \frac{1100-0}{\ln 1200/100} \ln \frac{R}{100} + 0 = 442 \ln \frac{R}{100}$$

| | | By Relaxation Method |
|---|---|---|
| @R = 300' | $P_1$ = 480 psi | $P_1$ = 476 psi |
| @R = 600' | $P_2$ = 784 psi | $P_2$ = 789 psi |
| @R = 900' | $P_3$ = 970 psi | $P_3$ = 970 psi |

Comparing actual values with values obtained by using the relaxation method, it is concluded that the relaxation method applied to flow of fluids will give fairly accurate results for the problem considered.

It must be noted that had other values of pressure been assumed, the results would have been slightly altered, as will be shown in problem No. 2.


Theoretical pressure
Relaxation pressure
Assumed pressure
Pressure in psi × 10²
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0
0 1 2 3 4 5 6 7 8 9 10 11 12
Drainage radius in feet × 10²
Plate I, problem I
Pressure distribution in radial flow
Given: boundary pressure 400 psi
radius 1200 feet
well pressure 0 psi
radius 100 feet

# PROBLEM 2

# THE RELAXATION METHOD OF MATHEMATICAL ANALYSIS

# APPLIED TO

# A MORE LENGTHY RADIAL FLOW PROBLEM

As a further illustration of the use of the relaxation method, consider the following two-dimensional radial flow problem.

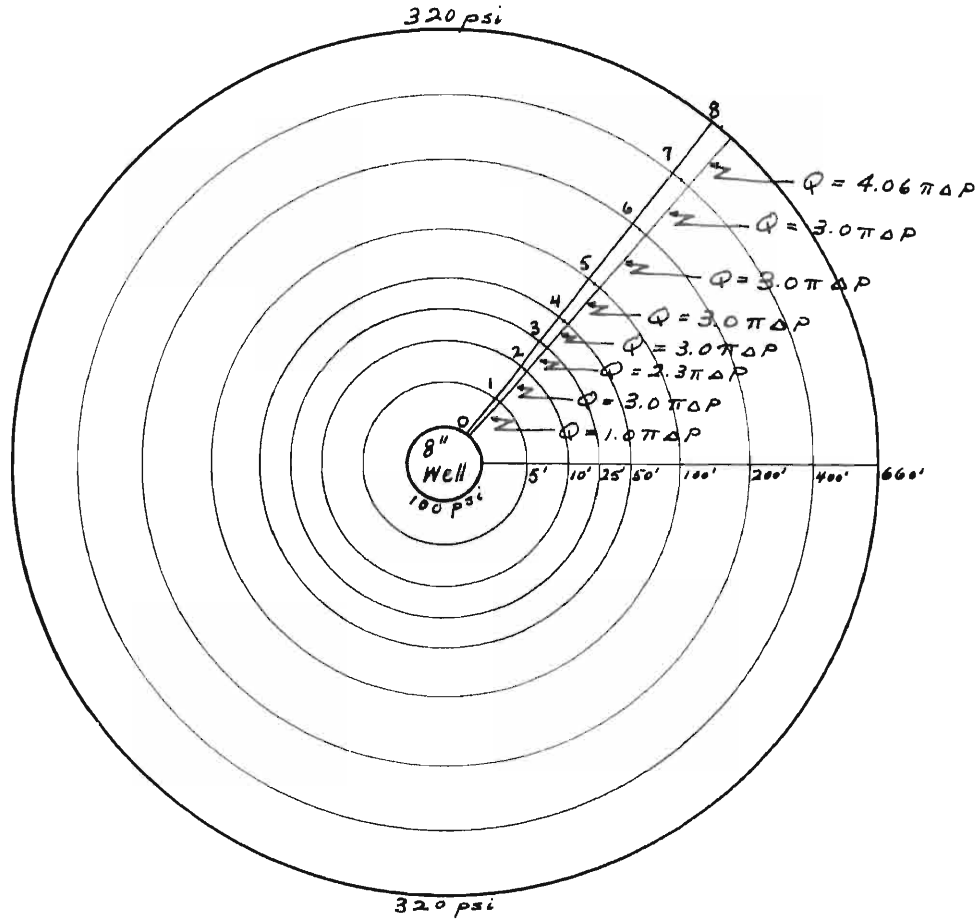



Fig. 2

Given well bore 8 inches
pressure 100 psi

drainage radius 660 feet
pressure 320 psi

Check pressure @ points 5', 10', 25', 50', 100', 200', and 400' from bore hole.

From D'Arcy's Law

$$Q = -\frac{K}{\mu} A \frac{\Delta P}{\Delta R}$$

where $\frac{k}{u} = 1$ (Assumed)

$A = 2\pi R$ (for unit thickness)

$$Q = \frac{2\pi R}{\Delta R} \Delta P$$

For particular problem

$$_{0}Q_{1} = {}_{1/3'}Q_{5'} = \frac{2\pi R}{\Delta R} \Delta P = \frac{2\pi(2.33)}{4.66} \Delta P = 1.0\,\pi \Delta P$$

$$_{1}Q_{2} = {}_{5'}Q_{10'} = \frac{2\pi(7.5)}{5} \Delta P = 3.0\,\pi \Delta P$$

$$_{2}Q_{3} = {}_{10'}Q_{25'} = \frac{2\pi(17.5)}{15} \Delta P = 2.3\,\pi \Delta P$$

$$_{3}Q_{4} = {}_{25'}Q_{50'} = \frac{2\pi(37.5)}{25} \Delta P = 3.0\,\pi \Delta P$$

$$_{4}Q_{5} = {}_{50'}Q_{100'} = \frac{2\pi(75)}{50} \Delta P = 3.0\,\pi \Delta P$$

$$_{5}Q_{6} = {}_{100'}Q_{200'} = \frac{2\pi(150)}{100} \Delta P = 3.0\,\pi \Delta P$$

$$_{6}Q_{7} = {}_{200'}Q_{400'} = \frac{2\pi(300)}{200} \Delta P = 3.0\,\pi \Delta P$$

$$_{7}Q_{8} = {}_{400'}Q_{660'} = \frac{2\pi(530)}{260} \Delta P = 4.06\,\pi \Delta P$$

As previously Q is + for source (flow into point)

and Q is - for sink (flow out of point)

From plate No. 2 assuming straight line variation of pressure with drainage radius,

| | |
|---|---|
| P @1/3' | 100 psi |
| P @5' | 101 psi |
| P @10' | 104 psi |
| P @25' | 108 psi |
| P @50' | 116 psi |
| P @100' | 132 psi |
| P @200' | 165 psi |
| P @400' | 232 psi |
| P @660' | 320 psi |

Results are shown in accompanying table No. 2 and plate No. 2.

It will be shown that if other than straight line variation of pressure with drainage radius is assumed, using the relaxation method to determine pressure, will result in very slight (negligible) or no change in final results, as is shown on table No. 4.

From plate No. 3 assuming curved line variation of pressure with drainage radius,

| | |
|---|---|
| P @1/3' | 100 psi |
| P @5' | 160 psi |
| P @10' | 200 psi |
| P @25' | 220 psi |
| P @50' | 240 psi |
| P @100' | 260 psi |
| P @200' | 280 psi |
| P @400' | 300 psi |
| P @660' | 320 psi |

Results are shown on table No. 4 and plate No. 3.

Again, the relaxation method applied to the preceding radial flow problem gives results of pressure very close or identical to those calculated from formula (1) for radial flow to a well, as shown below.

$$P = \frac{P_e - P_w}{\ln R_e/R_w} \ln \frac{R}{R_w} + P_w \qquad (1)$$

where $P_e = 320$ psi

$P_w = 100$ psi

$R_e = 660$ feet

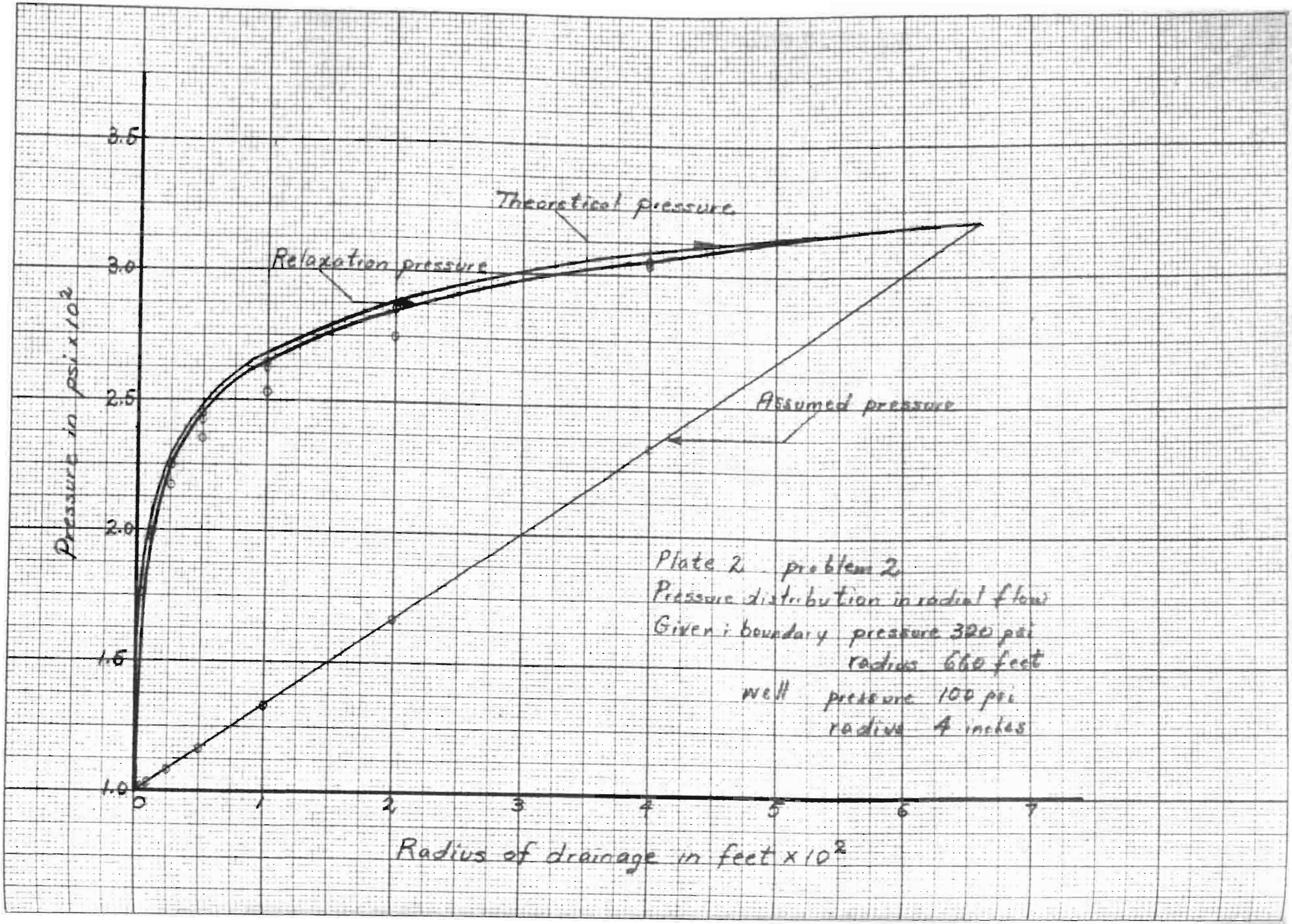
$R_w = 4$ inches $= 1/3$ feet

$$P = \frac{320-100}{\ln \frac{660}{1/3}} \ln \frac{R}{1/3} + 100 = 29 \ln 3R + 100$$

## Tabulated Results

| Step | $Q_0$ | $P_0$ | $Q_1$ | $P_1$ | $Q_2$ | $P_2$ | $Q_3$ | $P_3$ | $Q_4$ | $P_4$ | $Q_5$ | $P_5$ | $Q_6$ | $P_6$ | $Q_7$ | $P_7$ | $Q_8$ | $P_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | 100 | +8 | 101 | -2 | 104 | +17 | 108 | +24 | 116 | +51 | 132 | +102 | 165 | +156 | 232 | | 320 |
| 1 | | | | | | | | | | | | | +312 | | -338 | +70 | | |
| 2 | | | | | | | | | | | +381 | | -348 | +110 | -8 | | | |
| 3 | | | | | | | | | +384 | | -339 | +120 | +12 | | | | | |
| 4 | | | | | | | +377 | | -336 | +120 | +21 | | | | | | | |
| 5 | | | | | +251 | | -206 | +110 | -6 | | | | | | | | | |
| 6 | | | +293 | | -252 | +95 | +12 | | | | | | | | | | | |
| 7 | | | -7 | +75 | -27 | | | | | | | | | | | | | |
| 8 | | | | | | | | | +24 | | -39 | +10 | +42 | | | | | |
| 9 | | | | | | | | | | | -9 | | -18 | +10 | +22 | | | |
| 10 | | | | | | | +30 | | -12 | +6 | +9 | | | | | | | |
| 11 | | | | | | | | | | | | | -9 | | +1 | +3 | | |
| 12 | | | | | -11 | | -7 | +7 | +9 | | | | | | | | | |
| 13 | | | | | | | | | +15 | | -3 | +2 | -3 | | | | | |
| 14 | | | | | | | +2 | | -3 | +3 | +3 | | | | | | | |
| 15 | | | -16 | | +5 | -3 | -5 | | | | | | | | | | | |
| 16 | | | -4 | -3 | -4 | | | | | | | | | | | | | |
| 17 | | | | | | | | | +3 | | -3 | +1 | +3 | | | | | |
| 18 | | | | | | | -2 | | -3 | +1 | +3 | | | | | | | |
| Final Pressure | | 100psi | | 173psi | | 196psi | | 225psi | | 246psi | | 265psi | | 285psi | | 305psi | | 320psi |

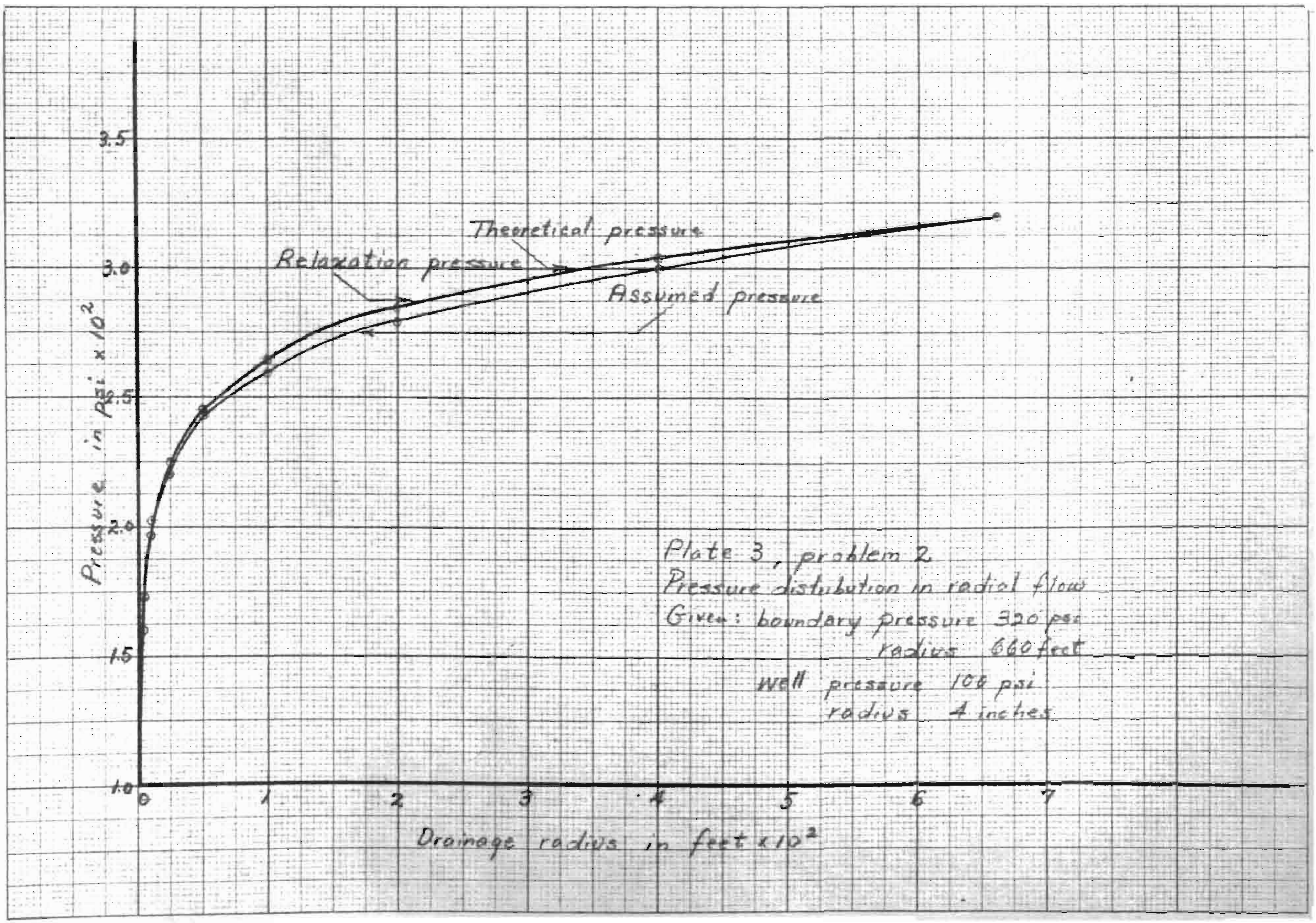
Table 2


Theoretical pressure
Relaxation pressure
Assumed pressure
Pressure in psi x 10²
Radius of drainage in feet x 10²
Plate 2 . problem 2
Pressure distribution in radial flow
Given : boundary pressure 320 psi
radius 660 feet
well pressure 100 psi
radius 4 inches

## Tabulated Results

| Step | $Q_0$ | $P_0$ | $Q_1$ | $P_1$ | $Q_2$ | $P_2$ | $Q_3$ | $P_3$ | $Q_4$ | $P_4$ | $Q_5$ | $P_5$ | $Q_6$ | $P_6$ | $Q_7$ | $P_7$ | $Q_8$ | $P_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | 100 | +60 | 160 | -74 | 200 | +14 | 220 | 0 | 240 | 0 | 260 | 0 | 280 | +21 | 300 | | 320 |
| 1 | | | -20 | +20 | -14 | | | | | | | | | | | | | |
| 2 | | | | | -5 | | -7 | +4 | +12 | | | | | | | | | |
| 3 | | | | | | | +2 | | -6 | +3 | +9 | | | | | | | |
| 4 | | | | | | | | | +6 | | -15 | +4 | +12 | | | | | |
| 5 | | | | | | | | | | | -6 | | -6 | +3 | +30 | | | |
| 6 | | | | | | | | | | | | | +9 | | -5 | +5 | | |
| 7 | | | | | | | | | | | 0 | | -3 | +2 | +1 | | | |
| 8 | | | | | | | +5 | | 0 | +1 | +3 | | | | | | | |
| 9 | | | | | +2 | | 0 | +1 | +3 | | | | | | | | | |
| 10 | | | 0 | -5 | -13 | | | | | | | | | | | | | |
| 11 | | | -9 | | +3 | -3 | -7 | | | | | | | | | | | |
| 12 | | | -1 | -2 | -3 | | | | | | | | | | | | | |
| 13 | | | | | -5 | | -1 | -1 | 0 | | | | | | | | | |
| Final Pressure | | 100psi | | 173psi | | 197psi | | 224psi | | 244psi | | 264psi | | 285psi | | 305psi | | 320psi |

Table 3


Theoretical pressure
Relaxation pressure
Assumed pressure
Plate 3, problem 2
Pressure distubution in radial flow
Given: boundary pressure 320 psi
radius 660 feet
well pressure 100 psi
radius 4 inches
Pressure in psi x 10²
Drainage radius in feet x 10²
3.5
3.0
2.5
2.0
1.5
1.0
0
1
2
3
4
5
6
7

Tabulated Pressure Values

| | Correct Method | Relaxation Method | |
|---|---|---|---|
| Radius | From formula | First assumption straight line variation | First assumption curved line variation |
| 5' | 177 psi | 173 psi | 173 psi |
| 10' | 198 psi | 196 psi | 197 psi |
| 25' | 225 psi | 225 psi | 224 psi |
| 50' | 245 psi | 246 psi | 244 psi |
| 100' | 265 psi | 265 psi | 264 psi |
| 200' | 285 psi | 285 psi | 285 psi |
| 400' | 305 psi | 305 psi | 305 psi |

Table 4

PROBLEM 3

SOLUTION OF SQUARE DRAINAGE AREA PROBLEM BY

THE RELAXATION METHOD OF MATHEMATICAL ANALYSIS

USING A SQUARE NETWORK OF FLOW

Statement of Problem

a.) Use rectangular network instead of radial as indicated below.

b.) Given 40 acre spacing or 1320 feet on each side.

c.) Pressure @ boundary 600 psi
Pressure @ sink 0 psi

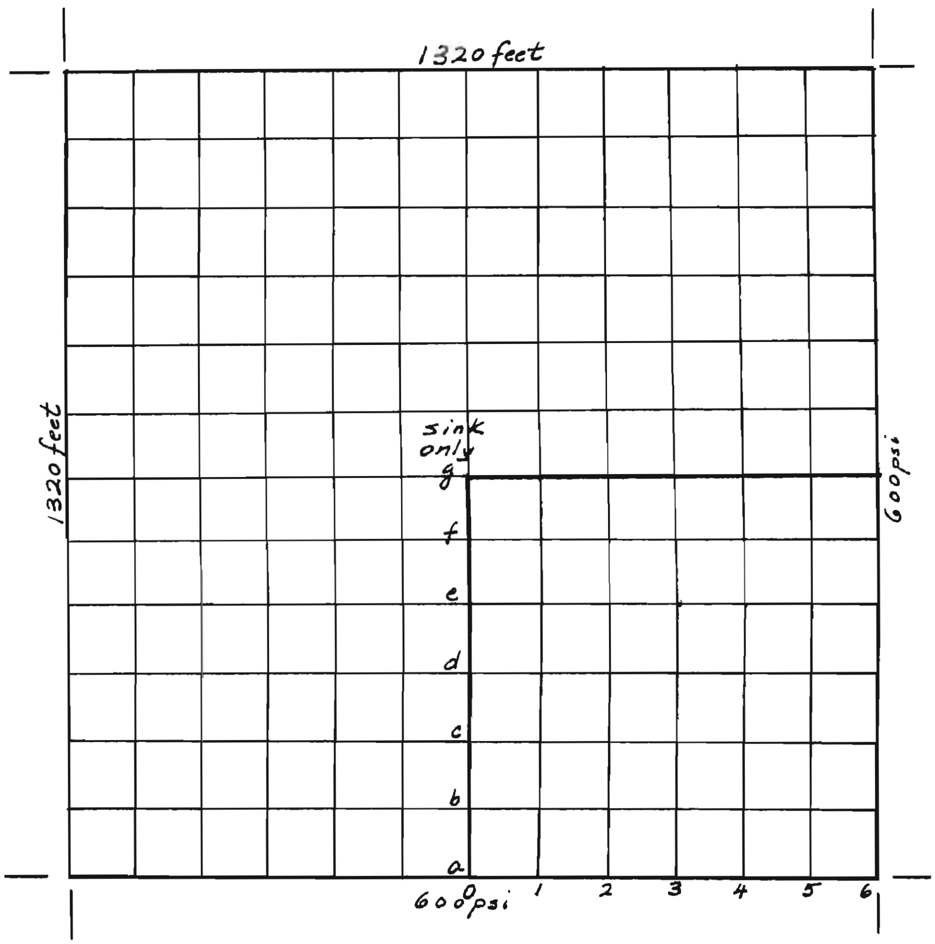



Fig. 3

Assumptions

1. a.) Since figure is symmetrical, to determine values of pressure at all points indicated (intersections of lines shown on figure 6), it will be only necessary to determine pressures at all points on lower right corner bounded by lines a-g and 0-6.

   b.) Again, since upper right part of cut figure is symmetrical to lower left part, pressures are calculated only at half of given points, e.g. point f,2 and e,1 are symmetrical to each other as are f,3 and d,1 - etc.

2. Consider all flow in any spacing, concentrated in a pipe shown as vertical and horizontal lines. For example: segment d,0 - d,1 denotes a pipe which includes flow from lower half of spacing d,0 - e,0 - e,1 - d,1 and upper half of segment c,0 - d,0 - d,2 - c,2.

3. Assume values of pressure for given points and consider horizontal and vertical flow, as shown by flow network.

4. Determine Q using assumed pressures, and if $Q \neq 0$ at particular point, relax by increasing or decreasing pressures until the value of Q equals or approaches very nearly to zero.

From D'Arcy's Law

$$Q = -\frac{K}{\mu} A \frac{\Delta P}{\Delta x}$$

where $\frac{k}{u}$ = 1 (Assumed)

$A$ = constant (same for all segments)

$\Delta x$ = constant (equal lengths of segments)

$\Delta P$ = difference in pressure between two points

$$Q = (Const.)\,\Delta P \;=\; f(\Delta P)$$

where

f = function of

## Sample Calculation

Determination of Q @ point e,3.

Step 0

From plate No. 3 assume pressures as follows:

@ e,3 p = 390 psi

@ e,4 p = 460 psi

@ d,3 p = 410 psi

@ e,2 p = 330 psi

@ f,3 p = 350 psi

$$Q_{e,3} = (P_{e,4}-P_{e,3}) + (P_{d,3}-P_{e,3}) + (P_{e,2}-P_{e,3}) + (P_{f,3}-P_{e,3})$$

$$= (460-390) + (410-390) + (330-390) + (350-390)$$

$$= -10 \text{ units}$$

All results in the order of calculation are shown on table No. 5.

Plates No. 4, No. 5 and No. 6 show plotting of pressure vs. distance in feet from sink.

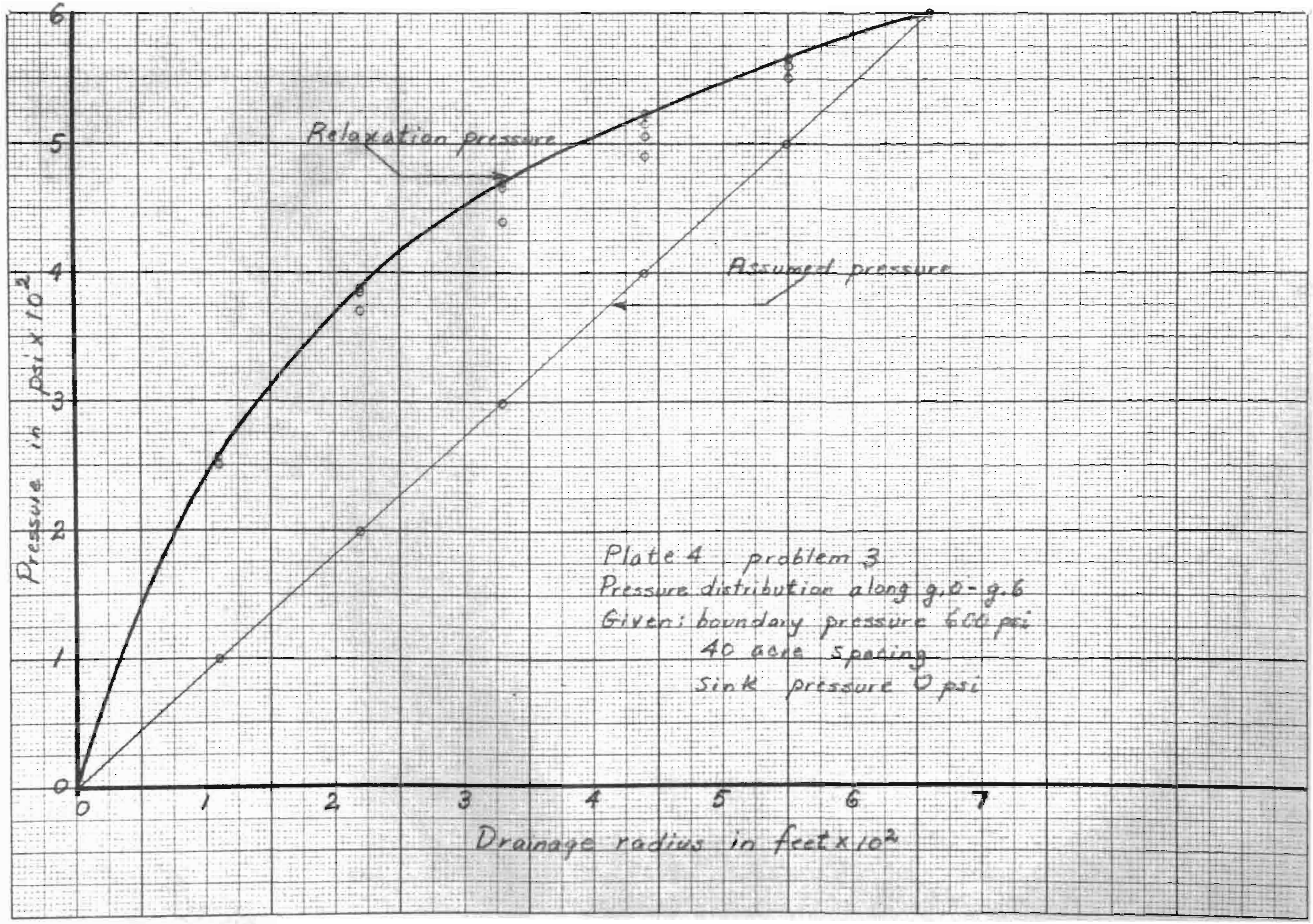
TABLE 5

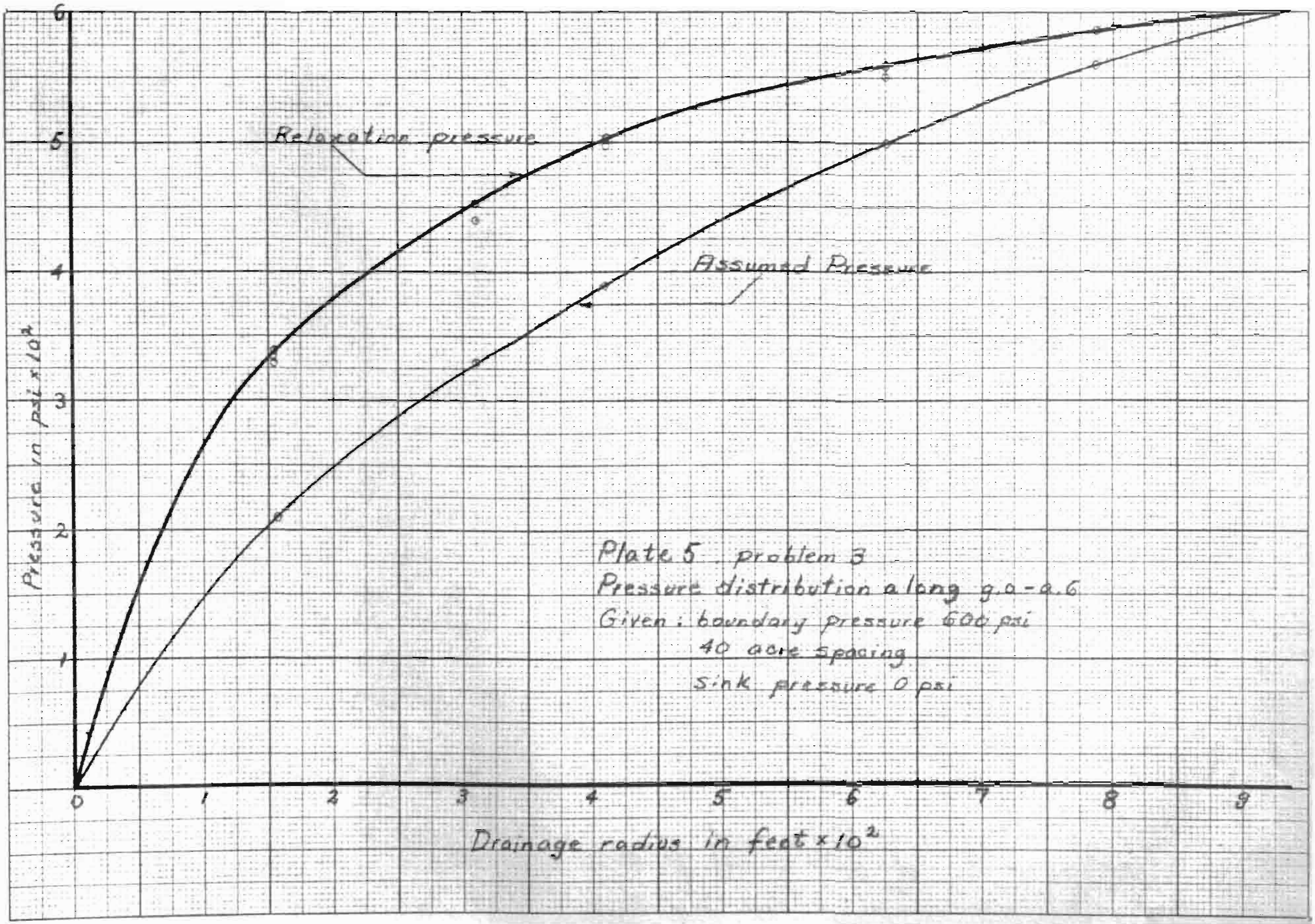
## Tabulated Results

| | $Q_{g,1}$ | $P_{g,1}$ | $Q_{g,2}$ | $P_{g,2}$ | $Q_{g,3}$ | $P_{g,3}$ | $Q_{g,4}$ | $P_{g,4}$ | $Q_{g,5}$ | $P_{g,5}$ | $Q_{f,1}$ | $P_{f,1}$ | $Q_{f,2}$ | $P_{f,2}$ | $Q_{f,3}$ | $P_{f,3}$ | $Q_{f,4}$ | $P_{f,4}$ | $Q_{f,5}$ | $P_{f,5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | +220 | 100 | +140 | 200 | +100 | 300 | +60 | 400 | +40 | 500 | -100 | 210 | +10 | 270 | -10 | 350 | +10 | 430 | -20 | 520 |
| | $-380^{1}$ | +150 | $+290^{1}$ | | $+270^{2}$ | | $+200^{3}$ | | $+130^{4}$ | | $+200^{1}$ | | $+180^{2}$ | | $+130^{3}$ | | $+100^{4}$ | | $+30^{5}$ | |
| | $-210^{2}$ | | $-390^{2}$ | +170 | $-290^{3}$ | +140 | $-160^{4}$ | +90 | $-70^{5}$ | +50 | $-280^{6}$ | +120 | $+300^{6}$ | | $+260^{7}$ | | $+200^{8}$ | | $+100^{9}$ | |
| | $+30^{6}$ | | $-250^{3}$ | | $-200^{4}$ | | $-110^{6}$ | | $-10^{14}$ | | $-20^{7}$ | | $-220^{7}$ | +130 | $-140^{8}$ | +100 | $-80^{9}$ | +70 | $-20^{10}$ | +30 |
| | $-30^{21}$ | +15 | $+10^{7}$ | | $0^{8}$ | | $+30^{9}$ | | $+5^{23}$ | | $+10^{21}$ | | $-120^{8}$ | | $-70^{9}$ | | $-50^{10}$ | | $+5^{14}$ | |
| | $-15^{22}$ | | $+25^{21}$ | | $+15^{22}$ | | $-30^{23}$ | +15 | $+25^{36}$ | | $+40^{27}$ | | $-10^{11}$ | | $+10^{12}$ | | $0^{13}$ | | $+15^{26}$ | |
| | $+5^{28}$ | | $-35^{22}$ | +15 | $+30^{23}$ | | $-15^{24}$ | | $-15^{38}$ | +10 | $0^{28}$ | +10 | $+5^{22}$ | | $+25^{24}$ | | $+15^{23}$ | | $+25^{35}$ | |
| | $-19^{40}$ | +6 | $-20^{24}$ | | $-30^{24}$ | +15 | $+5^{26}$ | | $-5^{39}$ | | $+12^{40}$ | | $+20^{25}$ | | $-35^{25}$ | +15 | $+30^{25}$ | | $-15^{36}$ | +10 |
| | $-14^{53}$ | | $+10^{27}$ | | $0^{25}$ | | $+25^{37}$ | | $-2^{57}$ | | $0^{54}$ | +3 | $-40^{27}$ | +15 | $-25^{26}$ | | $-10^{26}$ | +10 | $-5^{37}$ | |
| | $-8^{54}$ | | $+16^{40}$ | | $+10^{39}$ | | $+35^{38}$ | | $+4^{58}$ | | $+2^{55}$ | | $-30^{28}$ | | $-10^{27}$ | | $+5^{31}$ | | $+5^{38}$ | |
| | $-12^{55}$ | +1 | $+23^{52}$ | | $+26^{51}$ | | $-5^{39}$ | +10 | $+6^{76}$ | | $+4^{67}$ | | $-15^{30}$ | | $+5^{29}$ | | $+15^{36}$ | | $+10^{56}$ | |
| | $-10^{66}$ | | $+3^{53}$ | +5 | $-2^{52}$ | +7 | $+2^{52}$ | | $+2^{77}$ | +1 | $+6^{68}$ | | $-7^{51}$ | | $+15^{37}$ | | $-25^{37}$ | +10 | $-2^{58}$ | +3 |
| | $-14^{68}$ | +1 | $+4^{55}$ | | $+3^{53}$ | | $+12^{56}$ | | $+1^{87}$ | | $+2^{69}$ | +1 | $-2^{53}$ | | $+23^{41}$ | | $-15^{39}$ | | $0^{59}$ | |
| | $-12^{69}$ | | $+7^{62}$ | | $+6^{57}$ | | $0^{57}$ | +3 | | | $+4^{83}$ | | $+1^{54}$ | | $-9^{51}$ | +8 | $-8^{45}$ | | $+1^{72}$ | |
| | $-11^{84}$ | | $-1^{66}$ | +2 | $+12^{61}$ | | $+3^{62}$ | | | | $-2^{89}$ | | $+4^{61}$ | | $-2^{52}$ | | $0^{51}$ | | $+3^{75}$ | |
| | $+1^{89}$ | -3 | $+1^{67}$ | | $0^{62}$ | +3 | $+7^{75}$ | | | | $+2^{95}$ | -1 | $+6^{66}$ | | $+3^{56}$ | | $-20^{56}$ | +5 | $+4^{77}$ | |
| | $-1^{95}$ | | $+2^{68}$ | | $+2^{66}$ | | $-1^{76}$ | +2 | | | | | $+2^{67}$ | +1 | $+6^{57}$ | | $-17^{57}$ | | $+1^{86}$ | |
| | | | $+3^{81}$ | | $+4^{76}$ | | $0^{77}$ | | | | | | $+3^{69}$ | | $-6^{58}$ | +3 | $-14^{58}$ | | $+2^{92}$ | |
| | | | $+5^{83}$ | | $0^{81}$ | +1 | $+1^{81}$ | | | | | | $+5^{79}$ | | $-3^{61}$ | | $-11^{61}$ | | | |
| | | | $+1^{84}$ | +1 | $+2^{82}$ | | $-5^{86}$ | | | | | | $+6^{81}$ | | $-2^{63}$ | | $-9^{63}$ | | | |
| | | | $-2^{89}$ | | $+3^{84}$ | | $-1^{87}$ | -1 | | | | | $+2^{83}$ | +1 | $0^{74}$ | | $-7^{74}$ | | | |
| | | | | | $+2^{87}$ | | | | | | | | $+3^{84}$ | | $+2^{75}$ | | $-15^{75}$ | +2 | | |
| | | | | | $0^{88}$ | | | | | | | | $+2^{88}$ | | $+3^{76}$ | | $-13^{76}$ | | | |
| | | | | | | | | | | | | | $+1^{95}$ | | $-1^{82}$ | +1 | $-12^{82}$ | | | |
| | | | | | | | | | | | | | | | $0^{86}$ | | $0^{86}$ | -3 | | |
| | | | | | | | | | | | | | | | $-3^{87}$ | | $-1^{87}$ | | | |
| | | | | | | | | | | | | | | | $+1^{88}$ | -1 | $-2^{88}$ | | | |
| Final Pressure | 270 psi | | 393psi | | 466psi | | 519psi | | 561psi | | 343psi | | 417psi | | 476psi | | 524psi | | 563psi | |

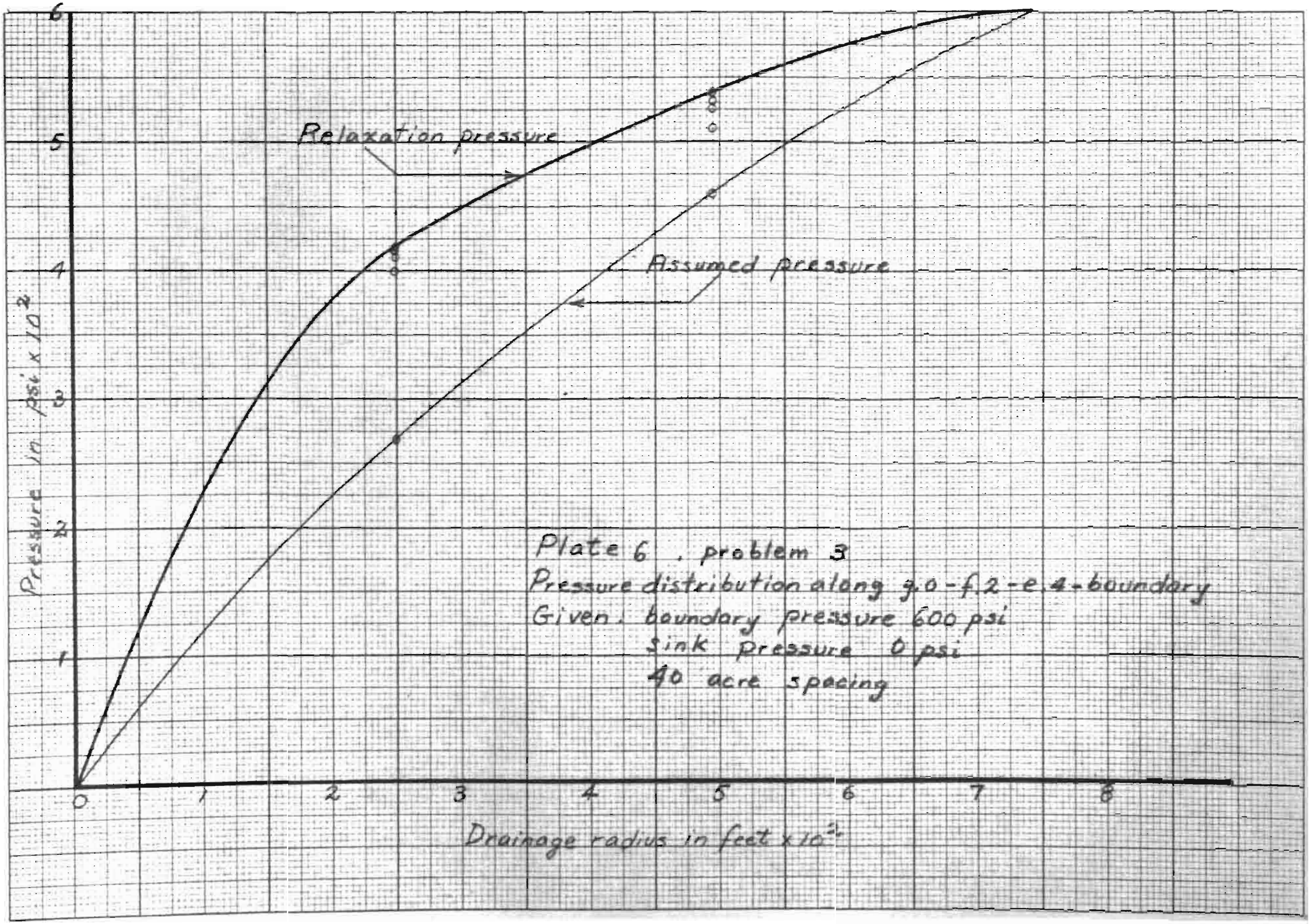
| | $Q_{e,2}$ | $P_{e,2}$ | $Q_{e,3}$ | $P_{e,3}$ | $Q_{e,4}$ | $P_{e,4}$ | $Q_{e,5}$ | $P_{e,5}$ | $Q_{d,3}$ | $P_{d,3}$ | $Q_{d,4}$ | $P_{d,4}$ | $Q_{d,5}$ | $P_{d,5}$ | $Q_{c,4}$ | $P_{c,4}$ | $Q_{c,5}$ | $P_{c,5}$ | $Q_{b,5}$ | $P_{b,5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 330 | -10 | 390 | 0 | 460 | 0 | 530 | +120 | 410 | -50 | 490 | +10 | 540 | +80 | 500 | 0 | 550 | +60 | 560 |
| | $+260^{7}$ | | $+90^{9}$ | | $+70^{9}$ | | $+30^{10}$ | | $+280^{12}$ | | $0^{13}$ | | $+35^{14}$ | | $+140^{16}$ | | $+20^{17}$ | | $+100^{19}$ | |
| | $-180^{11}$ | +110 | $+200^{11}$ | | $+150^{12}$ | | $+80^{13}$ | | $-80^{15}$ | +90 | $+90^{15}$ | | $+65^{16}$ | | $-60^{18}$ | +50 | $+70^{18}$ | | $0^{20}$ | +25 |
| | $-20^{12}$ | | $-120^{12}$ | +80 | $-50^{13}$ | +50 | $-20^{14}$ | +25 | $-20^{16}$ | | $-30^{16}$ | +30 | $-15^{17}$ | +20 | $-20^{19}$ | | $-10^{19}$ | +20 | $+20^{42}$ | |
| | $+10^{27}$ | | $-70^{13}$ | | $-25^{14}$ | | $0^{17}$ | | $+10^{29}$ | | $-10^{17}$ | | $+5^{19}$ | | $+20^{32}$ | | $+15^{20}$ | | $-4^{47}$ | +6 |
| | $+40^{29}$ | | $+20^{15}$ | | $+5^{16}$ | | $+15^{31}$ | | $+50^{32}$ | | $+40^{18}$ | | $+25^{32}$ | | $+40^{42}$ | | $+25^{34}$ | | $+2^{50}$ | |
| | $-20^{36}$ | +15 | $+35^{25}$ | | $+15^{26}$ | | $+25^{34}$ | | $-10^{33}$ | +15 | $+55^{31}$ | | $-15^{34}$ | +10 | $-8^{43}$ | +12 | $-15^{42}$ | +10 | $+4^{90}$ | |
| | $-4^{41}$ | | $-25^{29}$ | +15 | $+30^{29}$ | | $-15^{35}$ | +10 | $+6^{41}$ | | $-25^{32}$ | +20 | $-5^{35}$ | | $+12^{44}$ | | $-3^{43}$ | | $0^{91}$ | +1 |
| | $+2^{66}$ | | $-10^{30}$ | | $-30^{31}$ | +15 | $-5^{36}$ | | $+26^{44}$ | | $-10^{33}$ | | $+5^{42}$ | | $-4^{49}$ | +4 | $+3^{47}$ | | | |
| | $+4^{67}$ | | $+5^{31}$ | | $-10^{32}$ | | $+2^{45}$ | | $+6^{46}$ | +5 | $0^{34}$ | | $+15^{44}$ | | $+2^{60}$ | | $+7^{48}$ | | | |
| | $+8^{78}$ | | $+20^{33}$ | | $0^{35}$ | | $+6^{48}$ | | $+12^{60}$ | | $+12^{43}$ | | $-1^{48}$ | +4 | $+6^{64}$ | | $+11^{49}$ | | | |
| | $0^{79}$ | +2 | $-12^{41}$ | +8 | $+10^{37}$ | | $+9^{58}$ | | $+16^{64}$ | | $-28^{44}$ | +10 | $+2^{50}$ | | $-2^{70}$ | +2 | $-1^{50}$ | +3 | | |
| | $+2^{83}$ | | $-5^{45}$ | | $+18^{41}$ | | $+1^{59}$ | +2 | $+4^{65}$ | +3 | $-21^{45}$ | | $+4^{59}$ | | $+2^{71}$ | | $+1^{70}$ | | | |
| | | | $0^{46}$ | | $+28^{44}$ | | $+3^{63}$ | | $+8^{71}$ | | $-16^{46}$ | | $+6^{64}$ | | $-2^{85}$ | | $+3^{71}$ | | | |
| | | | $+8^{51}$ | | $0^{45}$ | +7 | $+5^{72}$ | | $+12^{78}$ | | $-12^{48}$ | | $+8^{71}$ | | $0^{90}$ | | $-1^{90}$ | +1 | | |
| | | | $-4^{60}$ | +3 | $+5^{56}$ | | $+1^{73}$ | +1 | $0^{80}$ | +3 | $-8^{49}$ | | $0^{72}$ | +2 | $-2^{93}$ | | $0^{91}$ | | | |
| | | | $-1^{61}$ | | $+7^{57}$ | | $+3^{74}$ | | $-4^{85}$ | | $-6^{63}$ | | $+1^{73}$ | | | | | | | |
| | | | $+1^{63}$ | | $+10^{60}$ | | $-1^{92}$ | +1 | $0^{86}$ | -1 | $-14^{64}$ | +2 | $-1^{85}$ | | | | | | | |
| | | | $+4^{65}$ | | $+2^{63}$ | +2 | | | $-2^{93}$ | | $-11^{65}$ | | $0^{90}$ | | | | | | | |
| | | | $+6^{74}$ | | $+4^{64}$ | | | | $+2^{94}$ | -1 | $-9^{70}$ | | $+1^{92}$ | | | | | | | |
| | | | $-2^{78}$ | +2 | $+6^{71}$ | | | | | | $-17^{71}$ | +2 | $0^{93}$ | | | | | | | |
| | | | $0^{79}$ | | $+7^{72}$ | | | | | | $-15^{72}$ | | | | | | | | | |
| | | | $+3^{80}$ | | $-1^{74}$ | +2 | | | | | $-13^{74}$ | | | | | | | | | |
| | | | $+4^{82}$ | | $+1^{75}$ | | | | | | $-10^{80}$ | | | | | | | | | |
| | | | $+3^{86}$ | | $+3^{78}$ | | | | | | $-2^{85}$ | -2 | | | | | | | | |
| | | | $+2^{88}$ | | $+1^{85}$ | | | | | | $-3^{86}$ | | | | | | | | | |
| | | | $+1^{94}$ | | $-2^{86}$ | | | | | | $+1^{93}$ | -1 | | | | | | | | |
| | | | | | $+1^{92}$ | | | | | | $0^{94}$ | | | | | | | | | |
| | | | | | $-2^{93}$ | | | | | | | | | | | | | | | |
| Final Pressure | 457psi | | 498psi | | 536psi | | 569psi | | 524psi | | 551psi | | 576psi | | 568psi | | 584psi | | 592psi | |

NOTE: small numbers to the immediate right of values of Q denote step number (order of calculation).


Pressure in psi x $10^2$
0
1
2
3
4
5
6
Drainage radius in feet x $10^2$
0
1
2
3
4
5
6
7
Relaxation pressure
Assumed pressure
Plate 4 problem 3
Pressure distribution along g.0 - g.6
Given: boundary pressure 600 psi
40 acre spacing
Sink pressure 0 psi



Relaxation pressure
Assumed Pressure
Plate 5 problem 3
Pressure distribution along g.a-a.6
Given: boundary pressure 600 psi
40 acre spacing
sink pressure 0 psi
Pressure in psi x 10²
Drainage radius in feet x 10²
0
1
2
3
4
5
6
7
8
9


Relaxation pressure
Assumed pressure
Plate 6 . problem 3
Pressure distribution along g.0 - f.2 - e.4 - boundary
Given: boundary pressure 600 psi
sink pressure 0 psi
40 acre spacing
Pressure in psi x 10^2
Drainage radius in feet x 10^2
0
1
2
3
4
5
6
7
8

## PROBLEM 4

## SOLUTION OF CIRCULAR DRAINAGE AREA PROBLEM BY THE RELAXATION METHOD OF MATHEMATICAL ANALYSIS USING A RECTANGULAR NETWORK OF FLOW

Finally consider the following problem involving vertical and horizontal flow, or a rectangular network applied to flow from a circular boundary to a sink.

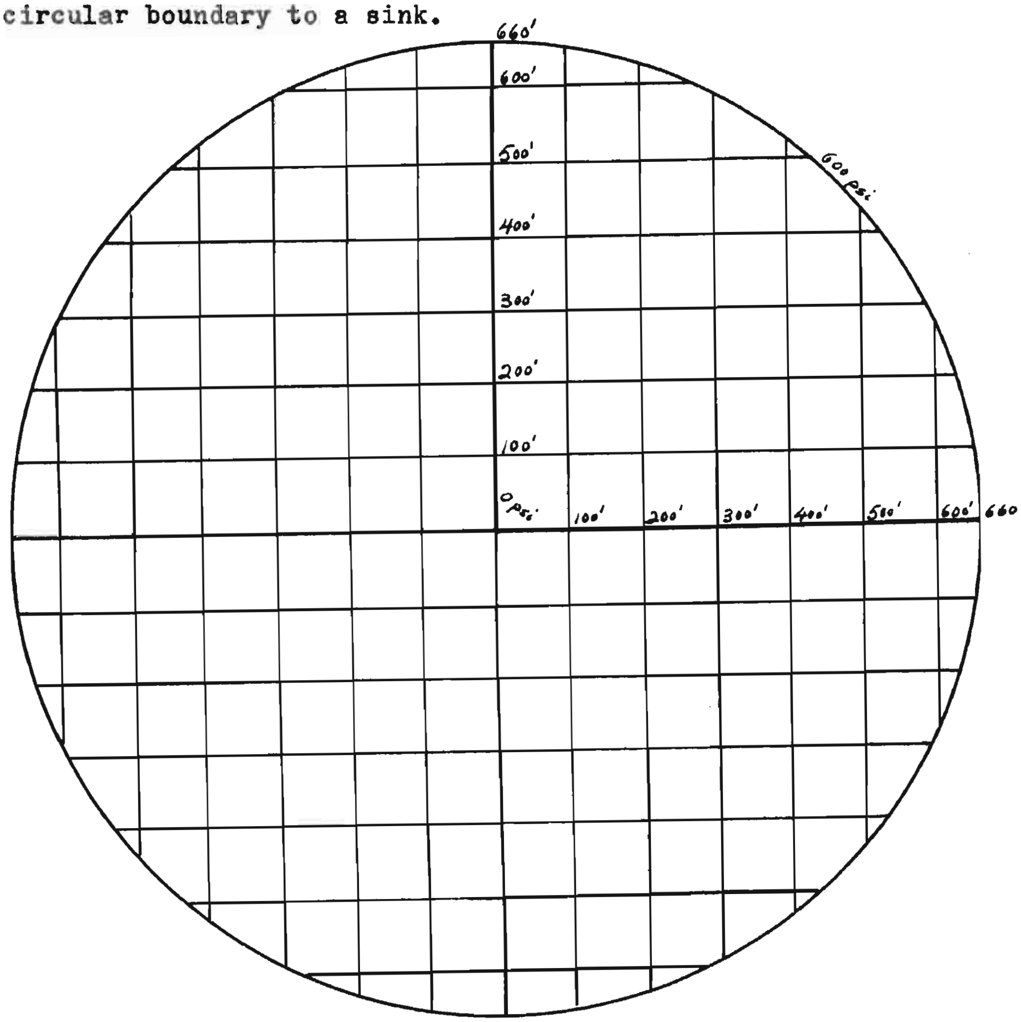



Fig. 4

Given: sink pressure 0 psi

drainage radius 660 feet
pressure 600 psi

To find pressures at all points indicated (intersections of vertical and horizontal flow lines).

Assumptions

1. Assume straight line variation of pressure with drainage radius at all points (shown on plate No. 7).
2. a.) Given circular figure is symmetrical, so consider only one quarter of circle as shown below.

   b.) Upper left corner is symmetrical to lower left, so consider only lower right corner.

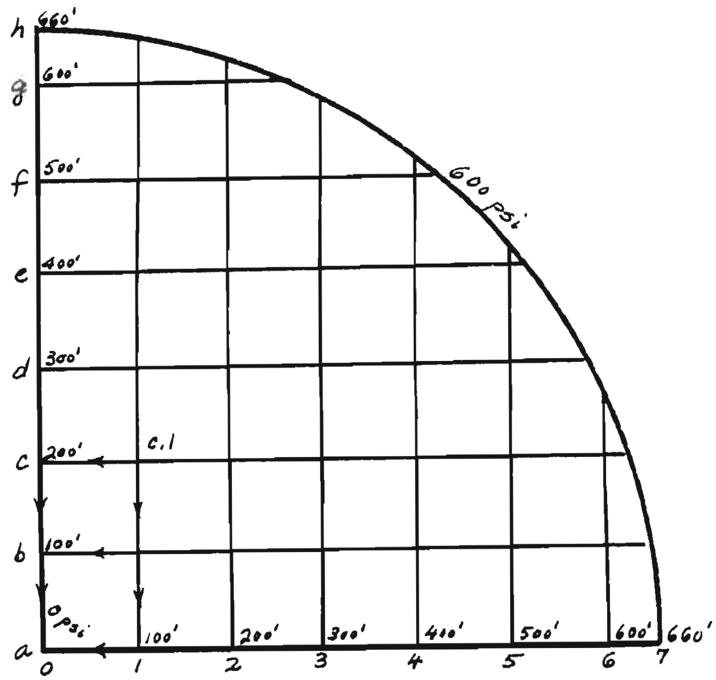



Fig. 5

3. Flow is as shown above in arrows e.g. for point c,1 to sink: direction and path of flow is along segments c,1 - c,0, c,0 - b,0, b,0 - a,0 and c,1 - b,1, b,1 - a,1, a,1 - a,0.

From D'Arcy's Law

$$Q = -\frac{k}{\mu} A \frac{\Delta P}{\Delta x}$$

where $\frac{k}{u}$ = 1 (Assumed)

A = constant (same for all segments)

$\Delta x$ = variable

= 100 feet for all segments excepting those segments indicated on figure 7.

$\Delta P$ = difference in pressure between two points where Q is considered.

$$Q = \frac{\Delta P}{\Delta x}$$

## Sample Calculation

Determination of Q @ point g,2

Step 0

From plates No. 7, No. 8 and No. 9 assumed pressures follow:

@ g,2 p = 575 psi

@ g,1 p = 553 psi

@ f,2 p = 490 psi

@ boundary p = 600 psi

From $Q = \frac{\Delta P}{\Delta x}$, it is noted that Q is inversely proportional to distance of flow $\Delta x$. For all points excepting those adjacent to the boundary, the length of path of flow is 100 feet. So we may assume it is a constant except for points in question.

When determining Q at point g,2, we note that vertical distance to boundary is 28 feet while horizontal distance to boundary is 75 feet.

Since we are not interested in units of Q, but in just its value, we may neglect entirely $\Delta x$ for all segments equaling 100 feet in length and use a proportionality constant in case length $\neq$ 100 feet.

e.g. at point g,2

$$Q = \frac{\Delta P}{\Delta x} = \left(\frac{600-575}{28}\right) + \left(\frac{600-575}{75}\right) + (490-575) + (553-575)$$

$= +16$ units

As before pressures are increased or decreased until all values of Q approach very nearly or are equal to zero. All results in the order of calculation are shown on table No. 6.

Plates No. 7, No. 8 and No. 9 show plotting of pressure vs. distance in feet from sink.

TABLE 6

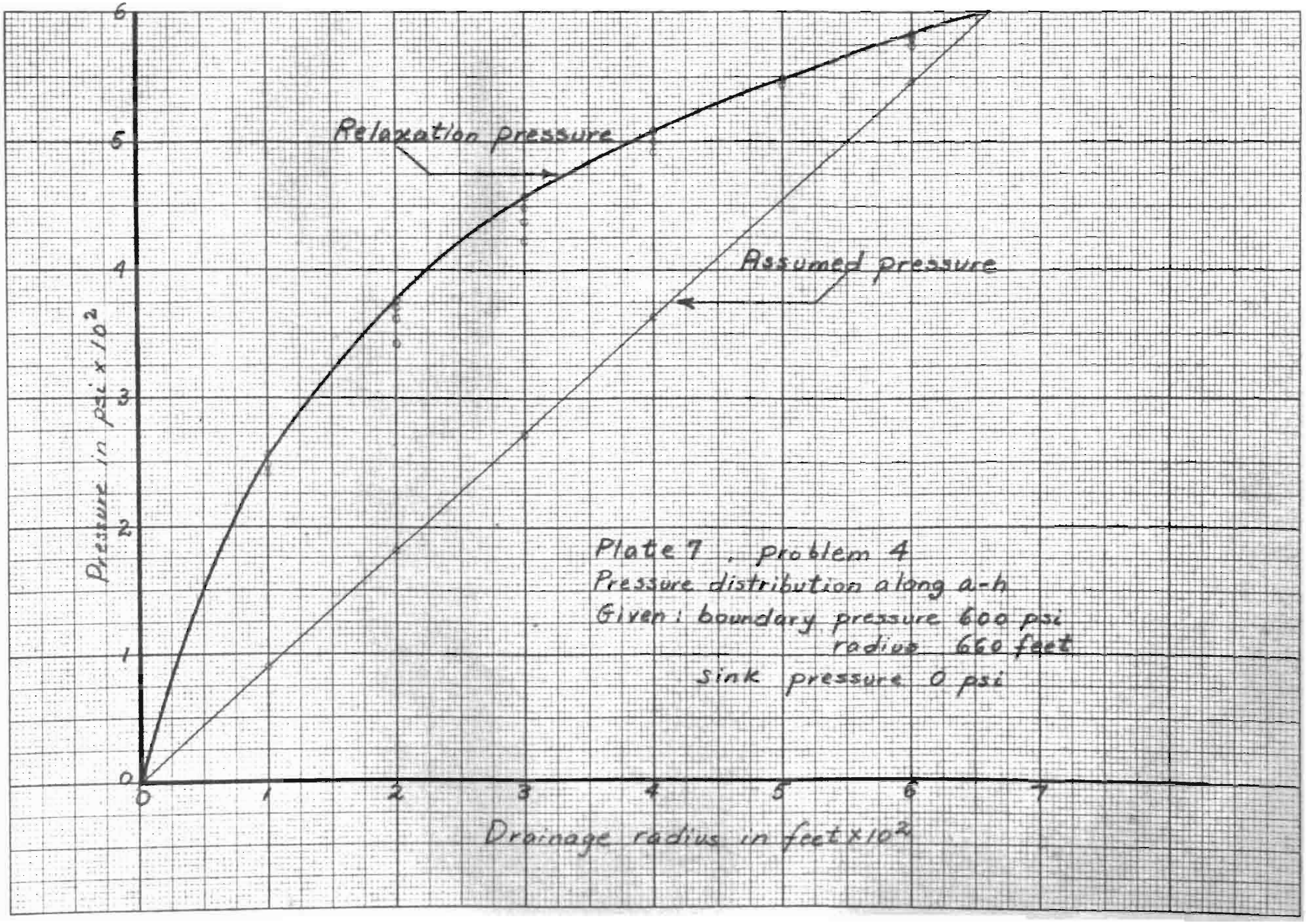
## Tabulated Results

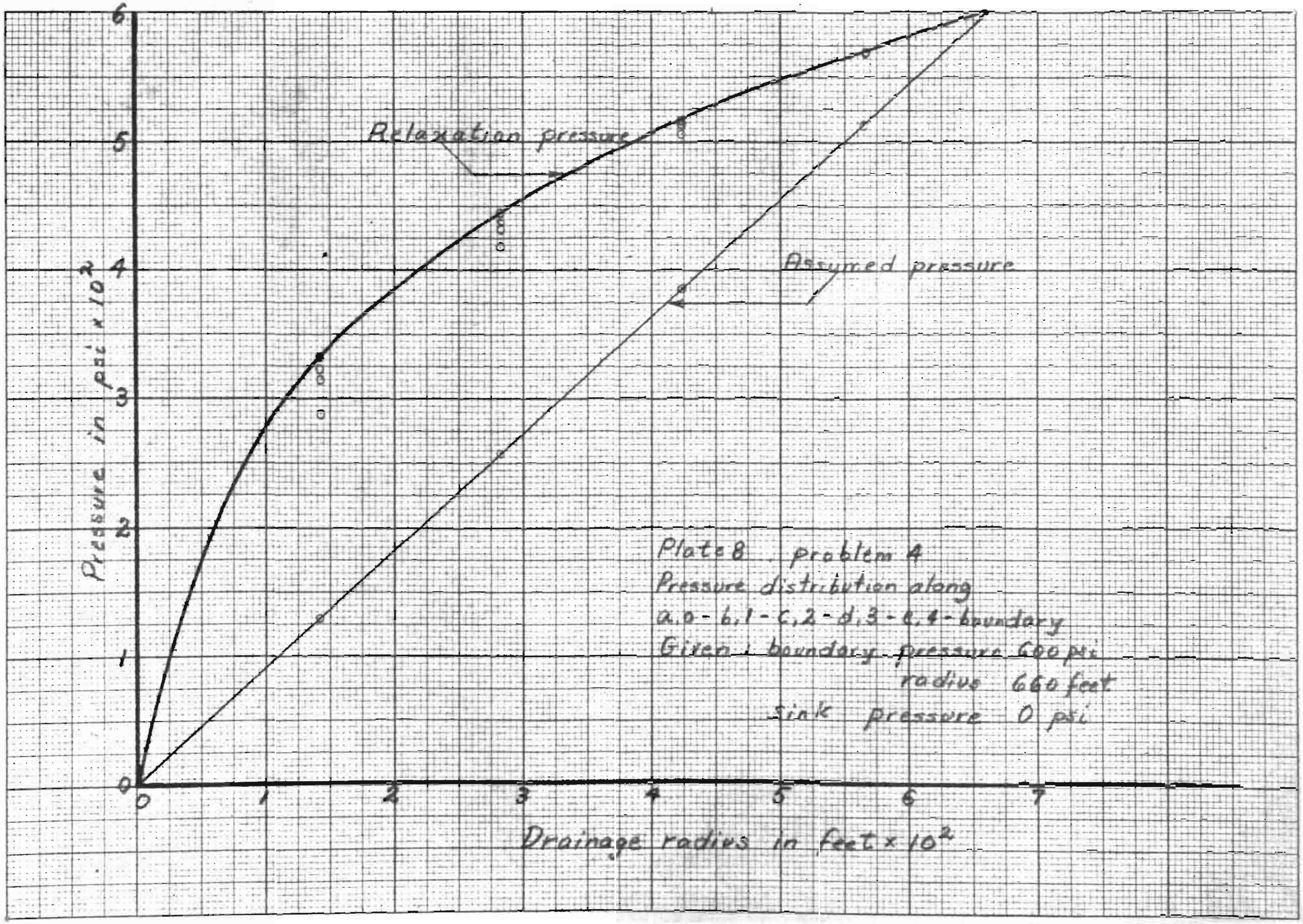
| $Q_{g,0}$ | $P_{g,0}$ | $Q_{g,1}$ | $P_{g,1}$ | $Q_{g,2}$ | $P_{g,2}$ | $Q_{f,0}$ | $P_{f,0}$ | $Q_{f,1}$ | $P_{f,1}$ | $Q_{f,2}$ | $P_{f,2}$ | $Q_{f,3}$ | $P_{f,3}$ | $Q_{f,4}$ | $P_{f,4}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +17 | 545 | +16 | 553 | +16 | 575 | +20 | 454 | +16 | 464 | +16 | 490 | +19 | 530 | +4 | 583 |
| $+107^{15}$ | | $+96^{16}$ | | $+86^{17}$ | | $+150^{10}$ | | $+141^{11}$ | | $+116^{12}$ | | $+109^{13}$ | | $+59^{14}$ | |
| $-33^{20}$ | +30 | $+125^{24}$ | | $+116^{21}$ | | $-210^{15}$ | +90 | $+231^{15}$ | | $+196^{16}$ | | $+178^{17}$ | | $+99^{18}$ | |
| $+27^{21}$ | | $-22^{21}$ | +30 | $+12^{22}$ | +15 | $-50^{16}$ | | $-89^{16}$ | +80 | $-84^{17}$ | +70 | $+13^{18}$ | +40 | $+5^{19}$ | +10 |
| $+3^{39}$ | +5 | $-7^{22}$ | | $-2^{61}$ | +2 | $-20^{20}$ | | $-19^{17}$ | | $-44^{18}$ | | $+22^{19}$ | | $+11^{41}$ | |
| $+6^{59}$ | | $-2^{39}$ | | $0^{62}$ | | $-12^{34}$ | | $+11^{21}$ | | $-29^{22}$ | | $-2^{41}$ | +6 | $+1^{48}$ | +1 |
| $-3^{60}$ | +2 | $+4^{40}$ | | | | $-7^{39}$ | | $+18^{35}$ | | $-11^{33}$ | | $-1^{48}$ | | $+2^{65}$ | |
| $+1^{62}$ | | $+6^{66}$ | | | | $+5^{40}$ | | $-6^{46}$ | +6 | $-5^{40}$ | | $0^{64}$ | | | |
| | | $+8^{61}$ | | | | $+9^{58}$ | | $-3^{57}$ | | $+1^{41}$ | | | | | |
| | | $-2^{62}$ | +2 | | | $-3^{59}$ | +3 | $0^{59}$ | | $+3^{61}$ | | | | | |
| | | | | | | $-1^{60}$ | | $+2^{62}$ | | | | | | | |
| Final Pressure 582psi | | 585psi | | 592psi | | 547psi | | 550psi | | 560psi | | 576psi | | 594psi | |

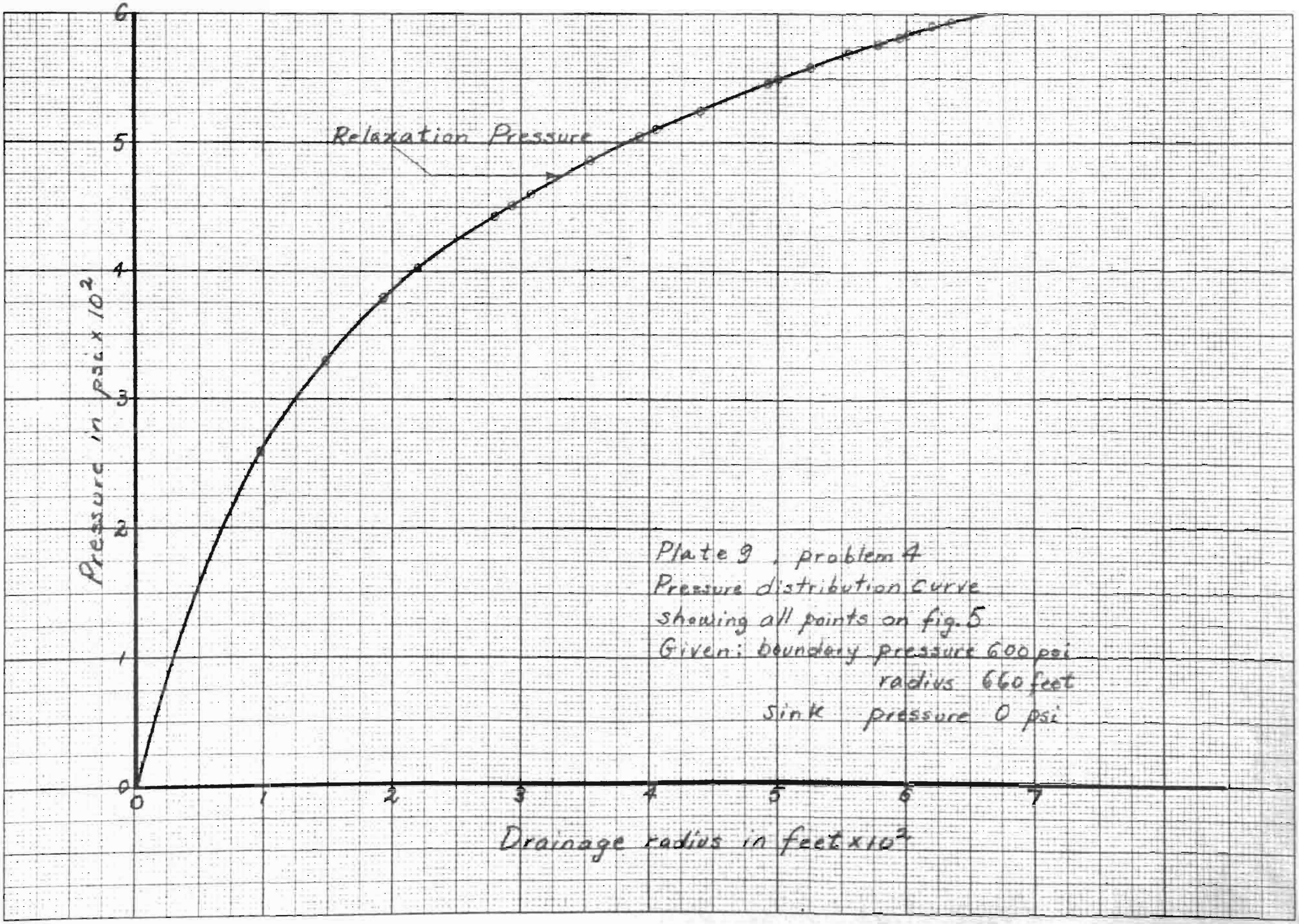
| $Q_{e,0}$ | $P_{e,0}$ | $Q_{e,1}$ | $P_{e,1}$ | $Q_{e,2}$ | $P_{e,2}$ | $Q_{e,3}$ | $P_{e,3}$ | $Q_{e,4}$ | $P_{e,4}$ | $Q_{d,0}$ | $P_{d,0}$ | $Q_{d,1}$ | $P_{d,1}$ | $Q_{d,2}$ | $P_{d,2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +25 | 363 | +21 | 375 | +20 | 407 | +17 | 455 | +20 | 514 | +27 | 273 | +32 | 287 | +26 | 328 |
| $+175^{6}$ | | $+171^{7}$ | | $+160^{8}$ | | $+137^{9}$ | | $+200^{13}$ | | $+187^{5}$ | | $+212^{4}$ | | $+186^{5}$ | |
| $-345^{10}$ | +130 | $+301^{10}$ | | $+285^{11}$ | | $+237^{12}$ | | $-20^{14}$ | +55 | $-413^{6}$ | +150 | $+362^{6}$ | | $+336^{7}$ | |
| $-95^{11}$ | | $-199^{11}$ | +125 | $-115^{12}$ | +100 | $-123^{13}$ | +90 | $0^{19}$ | | $-113^{7}$ | | $-238^{7}$ | +150 | $-224^{8}$ | +140 |
| $-5^{15}$ | | $-99^{12}$ | | $-25^{13}$ | | $-68^{14}$ | | $+2^{48}$ | | $+17^{10}$ | | $-98^{8}$ | | $-104^{9}$ | |
| $+10^{25}$ | | $-19^{16}$ | | $+45^{17}$ | | $-28^{18}$ | | $+6^{64}$ | | $+37^{24}$ | | $+27^{11}$ | | $-4^{12}$ | |
| $+20^{32}$ | | $-4^{26}$ | | $+57^{31}$ | | $-10^{33}$ | | $+2^{65}$ | +1 | $-23^{25}$ | +15 | $+42^{25}$ | | $+11^{26}$ | |
| $-12^{34}$ | +8 | $+14^{33}$ | | $-15^{33}$ | +18 | $-2^{36}$ | | | | $+7^{26}$ | | $-18^{26}$ | +15 | $+26^{27}$ | |
| $+2^{35}$ | | $+22^{34}$ | | $-8^{35}$ | | $+4^{41}$ | | | | $+15^{30}$ | | $-8^{29}$ | | $-22^{31}$ | +12 |
| $+4^{56}$ | | $-6^{35}$ | +7 | $-4^{44}$ | | $-4^{64}$ | +2 | | | $-25^{32}$ | +10 | $+4^{31}$ | | $-4^{33}$ | |
| $+10^{57}$ | | $0^{37}$ | | $-1^{57}$ | | $-3^{65}$ | | | | $-17^{34}$ | | $+14^{32}$ | | $+4^{36}$ | |
| $-6^{58}$ | +4 | $+6^{40}$ | | $0^{63}$ | | $-1^{72}$ | | | | $-5^{37}$ | | $+21^{35}$ | | $+10^{37}$ | |
| $-3^{59}$ | | $+9^{55}$ | | $+2^{64}$ | | | | | | $-1^{43}$ | | $-3^{37}$ | +6 | $+15^{42}$ | |
| $-1^{66}$ | | $-3^{57}$ | +3 | $+3^{73}$ | | | | | | $+1^{50}$ | | $+1^{44}$ | | $-1^{44}$ | +4 |
| | | $+1^{58}$ | | | | | | | | $+2^{54}$ | | $+6^{45}$ | | $+2^{49}$ | |
| | | $+2^{70}$ | | | | | | | | $+8^{65}$ | | $+9^{52}$ | | $+4^{53}$ | |
| | | | | | | | | | | $0^{56}$ | +2 | $-3^{55}$ | +3 | $+7^{55}$ | |
| | | | | | | | | | | $+4^{58}$ | | $-1^{56}$ | | $+3^{63}$ | +1 |
| | | | | | | | | | | $-4^{66}$ | +2 | $+2^{57}$ | | $+4^{70}$ | |
| | | | | | | | | | | $-2^{67}$ | | $+3^{63}$ | | $+6^{72}$ | |
| | | | | | | | | | | $0^{70}$ | | $+5^{66}$ | | $+2^{73}$ | +1 |
| | | | | | | | | | | | | $+1^{70}$ | +1 | $+3^{74}$ | |
| | | | | | | | | | | | | $+2^{71}$ | | | |
| | | | | | | | | | | | | $+3^{73}$ | | | |
| Final Pressure 505psi | | 510psi | | 525psi | | 547psi | | 570psi | | 452psi | | 462psi | | 486psi | |

| $Q_{d,3}$ | $P_{d,3}$ | $Q_{c,0}$ | $P_{c,0}$ | $Q_{c,1}$ | $P_{c,1}$ | $Q_{c,2}$ | $P_{c,2}$ | $Q_{b,0}$ | $P_{b,0}$ | $Q_{b,1}$ | $P_{b,1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| +22 | 386 | +44 | 182 | +40 | 204 | +32 | 258 | +76 | 91 | +74 | 129 |
| $+302^{8}$ | | $+194^{1}$ | | $+200^{2}$ | | $+392^{4}$ | | $-524^{1}$ | +150 | $+374^{1}$ | |
| $-178^{9}$ | +120 | $-446^{3}$ | +160 | $+360^{3}$ | | $-248^{5}$ | +160 | $-204^{2}$ | | $-266^{2}$ | +160 |
| $+2^{13}$ | | $-86^{4}$ | | $-360^{4}$ | +180 | $+32^{8}$ | | $-44^{3}$ | | $+94^{4}$ | |
| $+26^{31}$ | | $+64^{6}$ | | $-200^{5}$ | | $-28^{27}$ | +15 | $+6^{23}$ | | $-6^{23}$ | +25 |
| $-6^{36}$ | +8 | $-16^{24}$ | +20 | $-50^{7}$ | | $-8^{29}$ | | $+26^{24}$ | | $+10^{28}$ | |
| $+2^{44}$ | | $-1^{25}$ | | $-25^{23}$ | | $+16^{31}$ | | $-6^{28}$ | +8 | $+30^{29}$ | |
| $+4^{63}$ | | $+7^{28}$ | | $-5^{24}$ | | $-4^{42}$ | +5 | $+2^{30}$ | | $-2^{31}$ | +8 |
| $+8^{64}$ | | $+27^{29}$ | | $+10^{26}$ | | $+4^{44}$ | | $+18^{31}$ | | $+8^{38}$ | |
| $0^{72}$ | +2 | $-5^{30}$ | +8 | $+25^{27}$ | | $+14^{45}$ | | $-2^{38}$ | +5 | $+18^{45}$ | |
| $+2^{73}$ | | $+5^{32}$ | | $-15^{29}$ | +10 | $+2^{49}$ | +3 | $+2^{43}$ | | $-2^{46}$ | +5 |
| | | $+10^{38}$ | | $-7^{30}$ | | $+8^{52}$ | | $+12^{46}$ | | $+6^{47}$ | |
| | | $-6^{43}$ | +4 | $+1^{31}$ | | $0^{53}$ | +2 | $-4^{47}$ | +4 | $-2^{51}$ | +2 |
| | | $+4^{45}$ | | $+7^{37}$ | | $+2^{63}$ | | $-2^{50}$ | | $+4^{52}$ | |
| | | $+8^{47}$ | | $+12^{42}$ | | $+4^{71}$ | | $+2^{51}$ | | $-4^{68}$ | +2 |
| | | $0^{50}$ | +2 | $+16^{43}$ | | $+6^{73}$ | | $+3^{54}$ | | $0^{69}$ | |
| | | $+6^{52}$ | | $-4^{45}$ | +5 | $+2^{74}$ | +1 | $+5^{67}$ | | $+2^{71}$ | |
| | | $+2^{54}$ | +1 | $+1^{46}$ | | | | $+9^{68}$ | | | |
| | | $+4^{56}$ | | $+4^{49}$ | | | | $+1^{69}$ | +2 | | |
| | | $+6^{66}$ | | $+6^{50}$ | | | | | | | |
| | | $-2^{67}$ | +2 | $+8^{51}$ | | | | | | | |
| | | $0^{69}$ | | $-4^{52}$ | +3 | | | | | | |
| | | $+2^{71}$ | | $-2^{53}$ | | | | | | | |
| | | | | $-1^{54}$ | | | | | | | |
| | | | | $+2^{55}$ | | | | | | | |
| | | | | $+4^{67}$ | | | | | | | |
| | | | | $+6^{68}$ | | | | | | | |
| | | | | $+2^{70}$ | +1 | | | | | | |
| | | | | $+3^{71}$ | | | | | | | |
| | | | | $+4^{74}$ | | | | | | | |
| Final Pressure 516psi | | 379psi | | 403psi | | 444psi | | 260psi | | 331psi | |

NOTE: small numbers to the immediate right of values of Q denote step number (order of calculation).


Relaxation pressure
Assumed pressure
Plate 7 , problem 4
Pressure distribution along a-h
Given: boundary pressure 600 psi
radius 660 feet
sink pressure 0 psi
Pressure in psi x 10^2
Drainage radius in feet x 10^2
0
1
2
3
4
5
6
7


Relaxation pressure
Assumed pressure
Plate 8 . problem 4
Pressure distribution along
a,0 - b,1 - c,2 - d,3 - e,4 - boundary
Given : boundary pressure 600 psi
radius 660 feet
sink pressure 0 psi
Pressure in psi × 10²
Drainage radius in feet × 10²
0
1
2
3
4
5
6
7


Relaxation Pressure
Pressure in psi x 10^2
Drainage radius in feet x 10^2
0 1 2 3 4 5 6
0 1 2 3 4 5 6 7
Plate 9 , problem 4
Pressure distribution curve
showing all points on fig. 5
Given: boundary pressure 600 psi
radius 660 feet
sink pressure 0 psi

# CONCLUSIONS

From the results obtained in the solution of problems involving steady radial flow to a well, whose solutions are known, it may be seen that the mathematical analysis of relaxation can be successfully employed to the solution of similar problems.

Comparison of results obtained with those obtained by formula show negligible differences. It must be noted that the length of time to solve any given problem will depend primarily on two factors, namely: 1. values of pressure first assumed, and 2. degree and manner in which pressures are increased or decreased at particular points. This is clearly shown in the solution of problem No. 2.

From given results, it may be seen that the value of Q not always equals zero, even though the particular problem is considered completed. An attempt to bring all values of Q down to zero can be undertaken if more accurate values of pressure are required. Otherwise, depending on the given data, values of Q slightly greater than or less than zero will result in correct pressures with one or at most two pounds.

The solutions to problems No. 3 and No. 4 cannot be checked by formula, but they may be considered correct as 1. the curves drawn are quite satisfactory and 2. proof that the method will work, from problems No. 1 and No. 2.

Problem No. 4 tends to show that no matter what shape of drainage area one has, one can successfully apply the relaxation method of mathematical analysis assuming any desired form of network. The drainage area in problem No. 4 has a circular boundary, but a rectangular or rather a square network of flow was assumed and results were very satisfactory.

In conclusion it can be definitely stated that the mathematical analysis of relaxation will apply to the solution of problems involving steady flow of fluids through porous media.

## SUMMARY

A very brief review of the similarity between the solution of heat-conduction problems and that of flow of fluids through porous media is made.

The steps followed in the solution of a problem by use of the relaxation method are shown at the same time solving two radial flow problems which are checked for correctness using the formula for radial flow to a well.

Two more problems are solved using the relaxation method, one involving horizontal and vertical flow in a rectangular boundary, and the other a circular boundary.

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# VITA

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